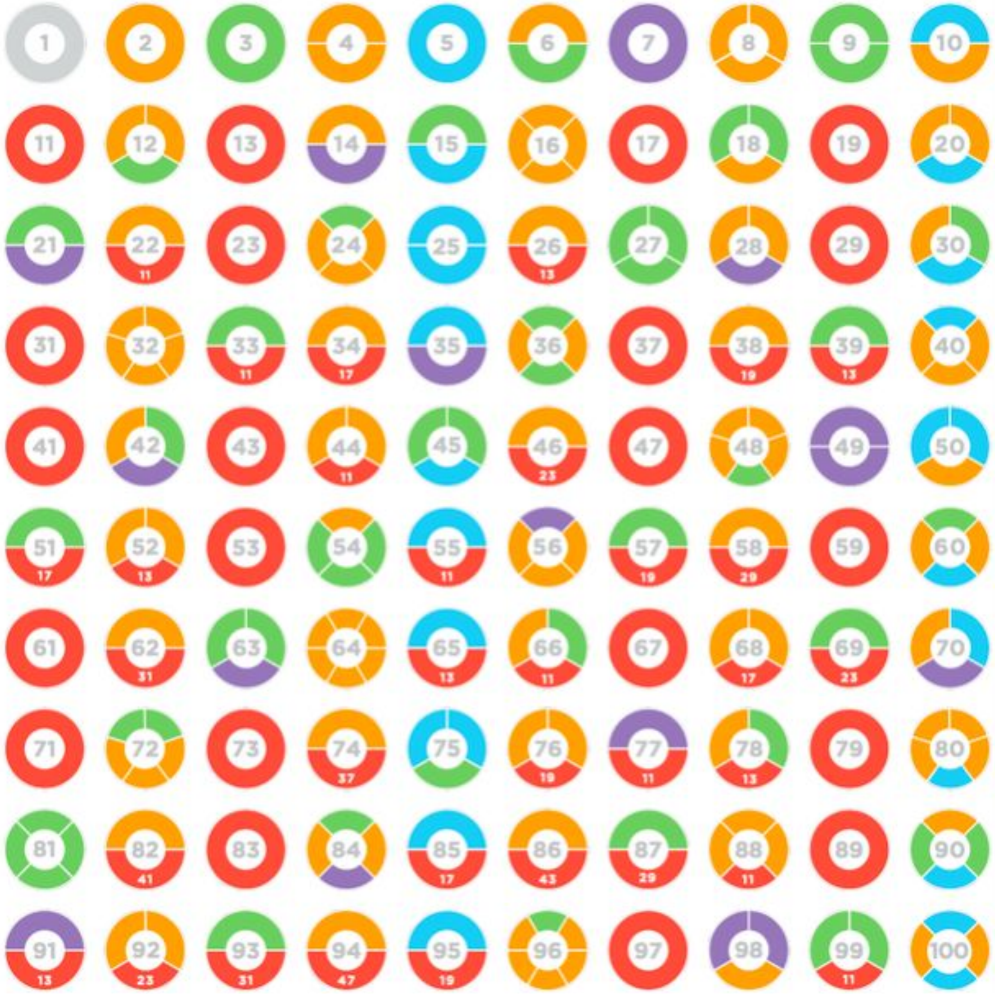
What do you notice?

What do you wonder?

Share any thoughts or observations in the chat.

The webinar will start soon.



IGCSE Mathematics: Developing a mathematical mindset through problem solving

Host Kaylene Connell

KOGNITY

“Absorb what is useful, reject what is useless, add what is essentially your own.”

- Bruce Lee

Webinar Overview

1. Mathematics in schools
2. The mathematical mindset.
3. A problem-based approach to mathematics and how it fits within IGCSE
4. What makes a good problem.
5. Strategies for using problem solving in the classroom.
6. Examples of problems and where to find them.
7. Where do we find the time?
8. Questions and answers.

Mathematics in Schools

Do we really believe that the math that most people are doing in school today is more than applying procedures to problems they don't really understand, for reasons they don't get?

- Wolfram TEDtalk, 2010

When a measure becomes a target it ceases to be a good measure

If exam results aren't the target then what is?

1. **Have fun!**

   Provide opportunities for students to enjoy maths.

2. **Build confidence**

   Not knowing the answer is the starting point.

3. **Increased depth of understanding**

   Allow students to struggle with problems and thus to build lasting connections in their brains.

4. **Linking**

   Provide opportunities to explore links within mathematics and between maths and other subjects.

IGCSE Aims

Aims

The aims describe the purposes of a course based on this syllabus.

The aims are to enable students to:

- develop mathematical skills and apply them to other subjects and to the real world
- develop methods of problem-solving
- interpret mathematical results and understand their significance
- develop patience and persistence in solving problems
- develop a positive attitude towards mathematics which encourages enjoyment, fosters confidence and promotes enquiry and further learning
- appreciate the elegance of mathematics

Aims

The aims describe the purposes of a course based on this syllabus.

The aims are to enable students to:

- develop an understanding of mathematical principles, concepts and methods in a way which encourages confidence, provides satisfaction and enjoyment, and develops a positive attitude towards mathematics
- develop a feel for number and understand the significance of the results obtained
- apply mathematics in everyday situations and develop an understanding of the part that mathematics plays in learners' own lives and the world around them
- analyse and solve problems, present the solutions clearly, and check and interpret the results
- recognise when and how a situation may be represented mathematically, identify and interpret relevant factors, select an appropriate mathematical method to solve the problem, and evaluate the method used

- Develop methods of problem-solving
- Develop patience and persistence in solving problems
- Appreciate the interdependence of different branches of mathematics

- Apply mathematics in everyday situations
- Analyze and solve problems
- Use mathematics as a means of communication with emphasis on ... structured argument

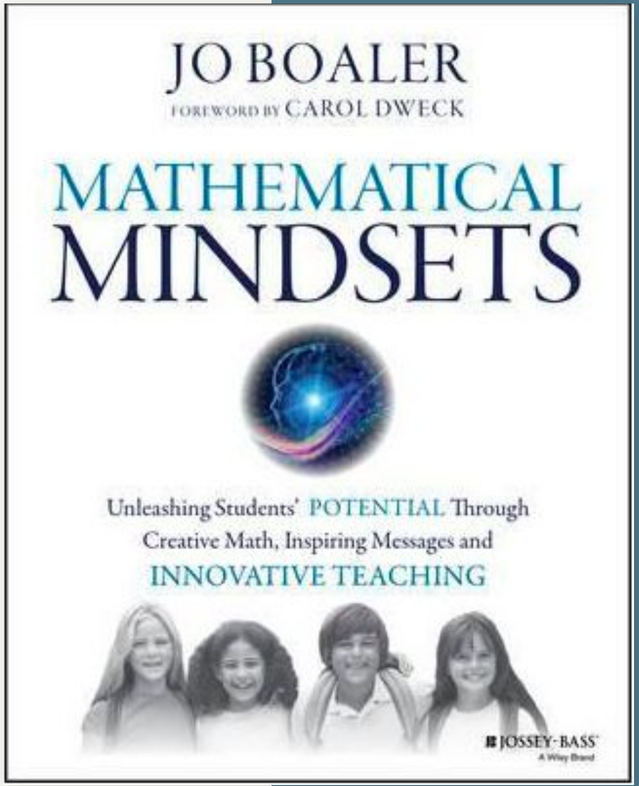
What about the unwritten aim?

“To help students score the best exam results they can.”

Research suggests that focusing on understanding can help us achieve this without us ***explicitly*** focusing on the exams.

What is a mathematical mindset?

- Practical strategies and activities that will help students of all ages discover that they can enjoy and succeed in maths.
- Research that backs the strategies.
- Great examples of problems.



Mathematicians collaborate!

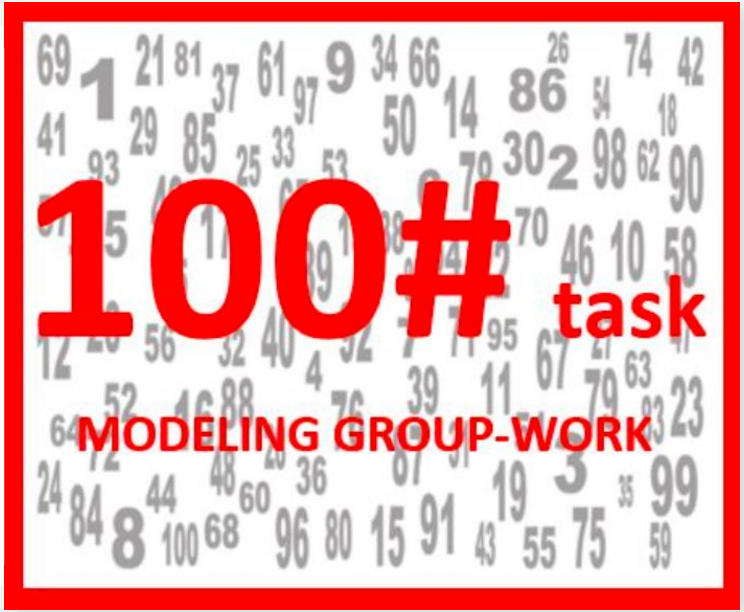
Good thinkers collaborate and communicate.

They work productively with other people, valuing different points of view. They are willing to change their mind when presented with convincing arguments. They know when to seek help, when to support others, when to speak up and when to compromise. (nrich)

The ability to look at a problem from different angles is crucial.
They [students] tend to look from one angle and you cannot see a way through. If you look at it from different angles, mysteriously perhaps something dawns on you and you find a way through. (Burton)

Collaborative Problem Solving

- Set up group work expectations early.
- Use group work regularly.
- A great way to do this is explained by Sara Van Der Werf
    - She now has an update for remote!



Type of problems

1. Conversation starters - get students talking, arguing justifying.
2. “Standard textbook/test questions with added depth - often created with a little argument and insistence on justification “convince me”
3. Students create questions themselves “what do you think, what do you wonder?” from photos/situations.
4. Patterns - either geometric, numeric or algebraic... “how do you see this change, does anyone see it differently”?
5. Problems with multiple answers
6. Extensions for students to go deeper instead of faster.
7. Technology based activities.

Some characteristics of good problems

- **Low floor/high ceiling** (Boaler - Mathematical Mindsets)
- Forcing students to justify, communicate and explain their answers - whether they are right or wrong.
- Rewarding deep thinking instead of speed.
- Did anyone see it in a different way?
- Fun and a sense of playfulness.

Where to find problems

Some of my favourite sources:

Books:

Mathematical Mindsets, Olympiad questions as a starting point. Kognity investigations.

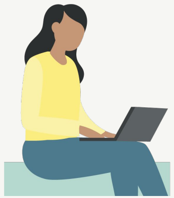
Websites:

Many amazing options - I'm going to share some of my favourites.

Exam papers:

Especially paper 5/6 from the 0607 syllabus but all sorts of exam questions can be deepened with the right questioning or a requirement on justification.

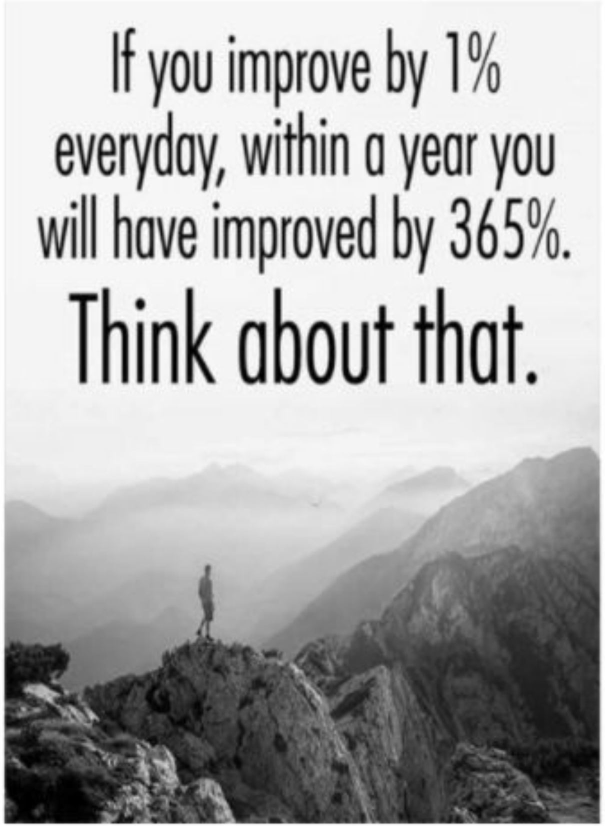
Social media



Your students

Conversation starters - get students talking, arguing justifying.

Convince me that this meme is true or false.



Standard textbook/test questions with added depth

- often created with a little argument and insistence on justification “convince me”

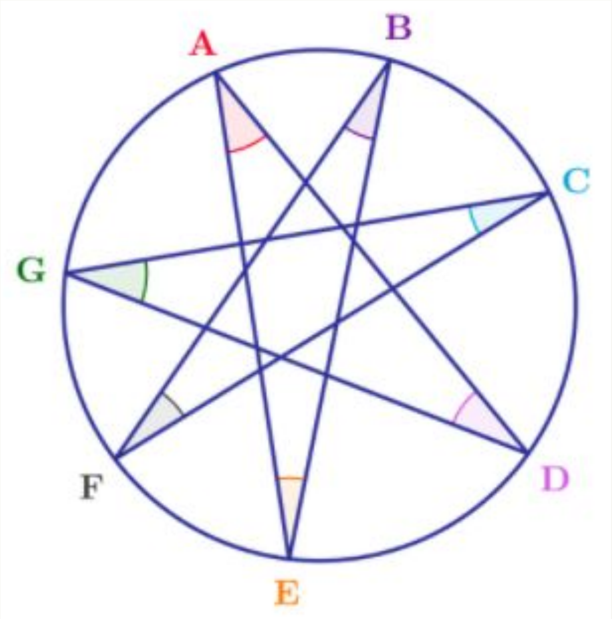
1. Write down a fraction which is equivalent to 3/5
2. Find the lowest common multiple (LCM) of 6 and 8
3. Without using your calculator work out $\frac{3}{4} \div 2\frac{1}{2}$.
4. Throw in a curveball!

5. An orchestra of 120 players takes 40 minutes to play Beethoven’s 9th Symphony. How long would it take for 60 players to play the symphony?

   Let P be number of players and T the time playing.

Patterns - geometric, numeric or algebraic...

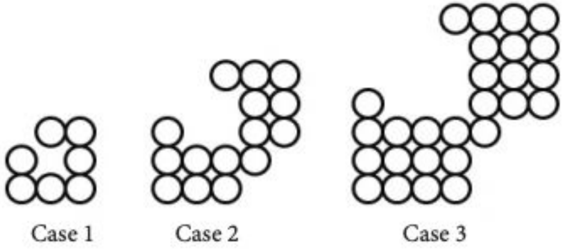
“how do you see this change, does anyone see it differently”?



0607 Investigation 5.3.5
0580 Investigation 4.3.6

youcubed at Stanford University

Circle Fever



1. How do you see the pattern growing? Where do you see the new circles being added when you move from case 1 to 2? Where do you see the new circles being added when you move from case 2 to 3?

2. What would the 10th case look like?

3. What would the 100th case look like? How many circles would be in the 100th case?

4. How many circles would be in the 0 case? What would it look like?

5. What would the -1 case look like?

6. Model the pattern with a rule or expression?

7. Write a rule for the number of circles in the 100th case?

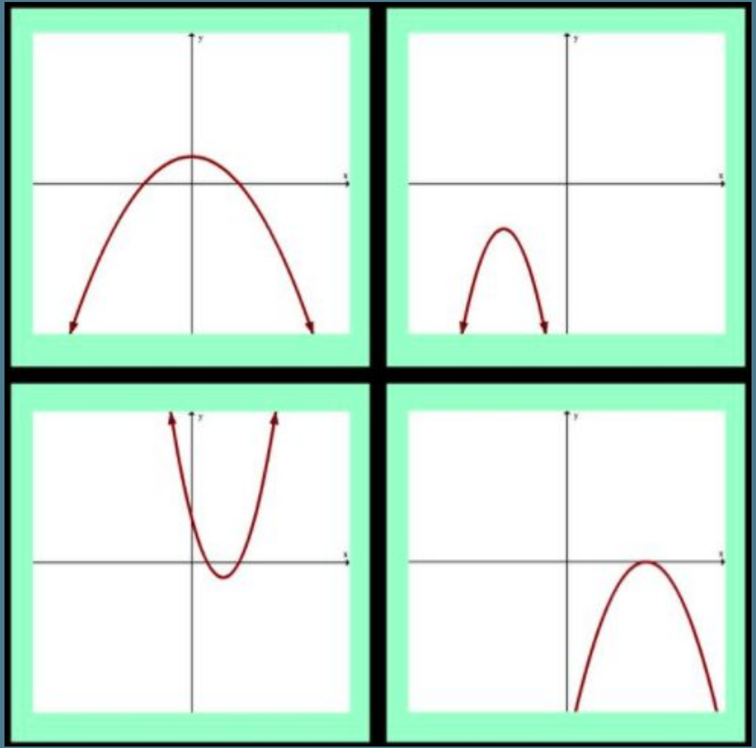
Problems with multiple answers

What are the next two numbers in this pattern?

2, 4, 6, ...

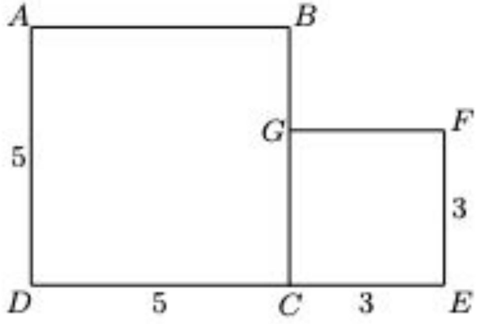
Can you come up with a different solution?

Which one doesn't belong?



Extensions for students to go deeper instead of faster.

1. A 5×5 square and a 3×3 square can be cut into pieces that will fit together to form a third square.
(a) Find the length of a side of the third square.
(b) In the diagram at right, mark P on *segment* DC so that $PD = 3$, then draw segments PA and PF. Calculate the lengths of these segments.
(c) Segments PA and PF divide the squares into pieces. Arrange the pieces to form the third square.



2. (Continuation) Change the sizes of the squares to $AD = 8$ and $EF = 4$, and redraw the diagram. Where should point P be marked this time? Form the third square again.

3. (Continuation) Will the preceding method *always* produce pieces that form a new square? If your answer is *yes*, prepare a written explanation. If your answer is *no*, provide a counterexample — two specific squares that can *not* be converted to a single square.

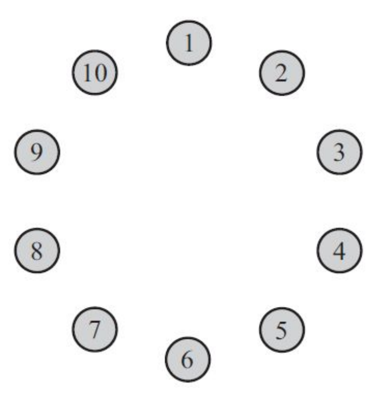
Paper 5/6 0607 (June 2009 P6)

There are 10 discs in a circle as shown. You remove the disc numbered 1, and, going clockwise, leave the next one, remove the one after that, leave the next one, and so on until only one disc remains.

If you've done it correctly the remaining disc will be number 4.

Test it out and make sure you understand how the rules work.

Start with a circle that has only 2 discs in it. Follow the same rules and record the number on the last disc remaining. Repeat with a circle that has 3 discs then 4 discs. Keep going until you think you have spotted a pattern. Make a hypothesis and then test it out.



Once you are confident about your pattern use it to predict the last number left if you have

a) 65 discs b) 125 discs c) 200 discs d) 100,000 discs

Convince me that your answers are correct.

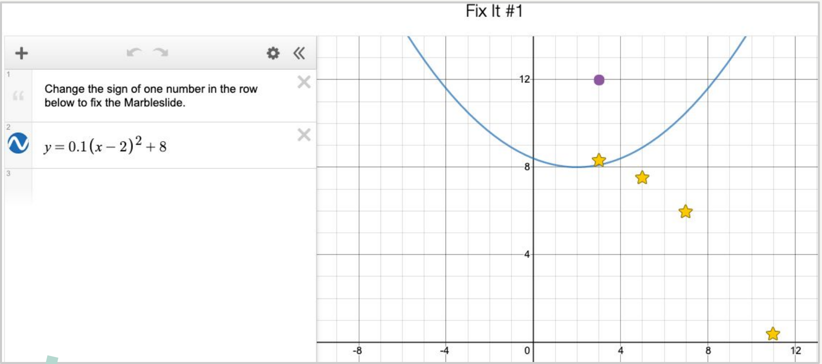
Find an algebraic expression for the number of discs in the circle, when the remaining disc is numbered 10.

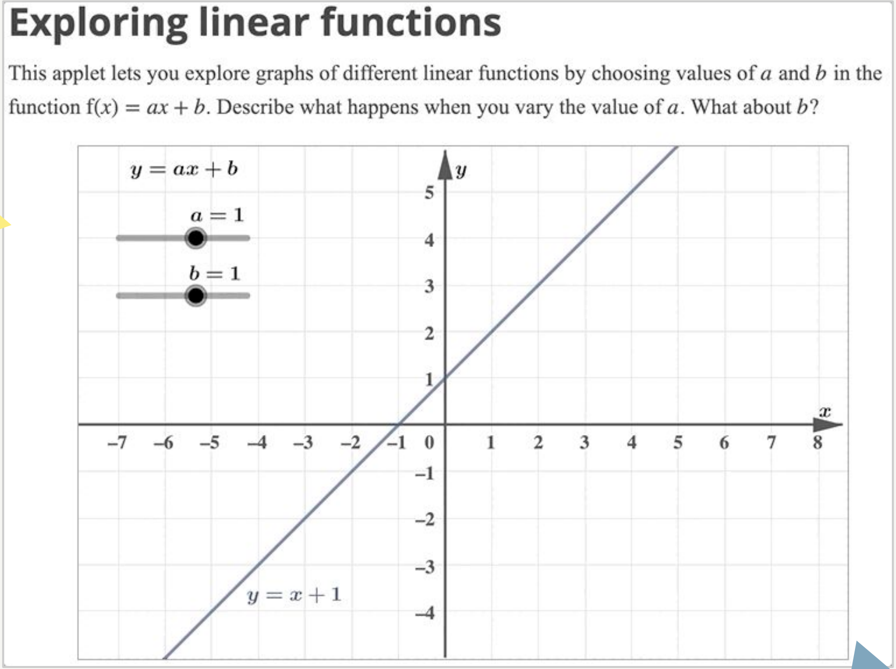
Write a sentence or two explaining the pattern that you used to figure this out.

Making use of technology

https://teacher.desmos.com/

Transformations of quadratics





Real problem solving involves frustration— temporary, we hope—as students explore different routes to a solution. Students need to develop perseverance in problem solving.

Thinking happens only when we have time to struggle.

As usual time is the enemy, and projects take time.

- Time for you to find problems
- Time for students to think deeply, make mistakes, wrestle with the issues
- Time for both you and them to reflect on what was learned

Thanks and Questions

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